Life Cycle Uncertainty and Portfolio Choice Puzzles

Yongsung Chang
University of Rochester
Yonsei University

Jay H. Hong
University of Rochester

Marios Karabarbounis
*University of Rochester**

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Abstract

The standard portfolio-choice theory is hard ro reconcile with the following facts. i) Despite a high rate of returns the average household holds a low share of risky share (equity premium puzzle). ii) The share of risky assets is disproportionately larger for richer households. iii) The share of risky assets increases in age. We show that a simple life-cycle model with Bayesian learning about earnings ability can account for all three facts. Younger-wealth poor households whose incomes are skewed toward labor earnings face a large amount of risk. To hedge risk they invest in safe financial assets. As households grow older their willingness to invest in risky assets increases partially because their ability is gradually revealed and also because a larger amount of accumulated assets decreases their total risk exposure.

*Emails: *yongsung.chang@rochester.edu*, *jay.hong@rochester.edu* and *mkarampa@z.rochester.edu*

1 Introduction

Portfolio choice, the allocation of saving between riskless and risky accounts has attracted a lot of attention in the last decade especially because the literature has identified several persisting puzzles. According to Haliassos and Michaelides (2002) there are three empirical facts that is hard to explain: 1) The limited participation in the stock market and the low share of risky investment for people who do participate, in spite of the high equity premium. 2) The positive relation between financial wealth and the risky share of financial assets conditional on participation. 3) The weakly increasing or relatively stable relation between risky share and age again conditioning on those who hold at least one risky account. The last fact is even more striking given that financial planners advise people to shift their portfolios towards less risky assets at later ages of the life cycle.

This paper is an attempt to uncover the driving forces of portfolio behavior and address the long standing puzzles. Our central focus is the tight link between wage risk and investment behavior. As discussed in a number of papers (Jaganathan and Korcelakota 1997, Cocco, Gomes and Maenhout 2005) labor income is a (non-tradable) asset with its own risk characteristics. Workers who face small labor income uncertainty can afford taking more risk in the financial market since their total income risk is relatively low. On the other hand, workers with uncertain job market prospects are more conservative in their investment decisions. So understanding the nature, amount and variation in wage risk is key to explaining portfolio choice behavior.

We shed light into this issue by considering a life cycle, portfolio choice model in the spirit of Cocco, Gomes and Maenhout (2005). We depart from the literature by making the assumption that workers cannot identify the specific components that drive the wage realization. Mincerian wage regressions decompose wage differentials across observables like education and age which determine the position and slope of individual wage profiles and residual factors which account for fluctuations along these profiles. A growing literature (Baker 1997, Guvenen 2008, Guvenen and Smith 2010, Huggett et al. 2010) provides evidence that there is significant heterogeneity in individual wage profiles. In this case it is sensible to assume that individuals cannot predict the “observable" components at the time of labor market entry but gradually learn them through successive wage realizations. We show how this framework can address a wide range of puzzles including the age-profile of the risky share, the correlation between risky share and financial assets and last but not least the equity premium puzzle. This takes place through two channels.

First, in the presence of imperfect information the individual *perceived* wage risk is much larger than what residual variations in wages might suggest. This reduces the willingness to invest in risky assets and can explain why the equity premium is so high. Second, wage risk is much smaller for older investors partially because their (present value) of life-time labor income is smaller and also due to their information update. As a result older investors can afford taking more risk in the financial market as the data suggest.

We quantitatively test these ideas by incorporating a Bayesian learning model into a life cycle model of portfolio choice. As in a standard incomplete markets model workers receive random wage draws every period. Although full insurance is not possible they can partially insure using two savings instruments. A risk free bond and a stock which pays on average more to convince investors to bear the extra financial risk. The bond corresponds to assets of low riskiness (checking accounts, savings accounts etc.) while the stock corresponds to assets of high riskiness (stocks, stock funds etc.). The wage is a function of several components: a fixed effect (ability), a life cycle component (wage growth), an idiosyncratic shock of some persistence and an i.i.d. component. While agents observe their wages they cannot understand which component drives the wage realization. Simply put, a worker cannot understand if her wage is high because she is good or because she is just lucky. Instead she forms beliefs about her quality and update these beliefs in a Bayesian fashion using the wage realization as a signal. To generate an active extensive margin we assume that stock market participation requires a monetary payment. The model is a finite horizon model of 60 periods and is solved in partial equilibrium. Workers enter the labor market at the age of 21 and work up to the age of 65. Once they enter retirement agents receive a retirement benefit.

The model is calibrated to match key features of the US economy. The returns to the two savings instruments, bonds and stocks match the empirical equity premium and risk free rate of Treasury bonds. We assume CRRA preferences with a coefficient or risk aversion of 6. We parameterize the discount factor to match the empirical wealth to labor income ratio. Reproducing a realistic asset holdings distribution is crucial to matching the average risky share. Heterogeneity in wages is expressed through life cycle observables (fixed effect and slope of wage profile) and random shocks. The latter component is modeled as an AR(1) process plus some noise. We calibrate the labor income variance to match the empirical variance of log-hourly wages from the PSID.

We evaluate the ability of our imperfect information model (IIM) to account for the

basic patterns observed in the data by comparing its implications to a model of perfect information (PIM). For both models we generate key statistics like the participation rate, the average risky share, the correlation of risky share with total financial assets and the age-profile of the risky share. We compare both models to statistics calculated from the Survey of Consumer Finances for 1998. In line with the portfolio choice literature we find that the PIM fails remarkably to account for the basic patterns observed in the data. In sharp contrast the IIM can address jointly all puzzles. The average risky share is **0.47** in the data **0.75** in the PIM and **0.47** in the IIM. The correlation of financial assets with the risky share conditional on participation is **0.14** in the data **-0.33** in the PIM and **0.12** in the IIM. Lastly the IIM generates an increasing profile for the risky share while the PIM predicts that younger investors should hold relatively more stocks. The average growth rate of the risky share for people who hold at least one risky account is **0.4%** in the data **-0.8%** in the PIM and **0.2%** in the IIM. We show that the IIM tracks pretty closely the investment behavior along the life cycle. We find the imperfect information model to have a high added value mainly to the conditional (on participation) moments.

The reason the IIM explains very well the portfolio composition matches the data better has to do with our assumptions about wage risk. Under imperfect information: 1) The willingness to take equity risk decreases significantly. Large labor income risk crowds out financial risk. Since consumption is much more volatile investors wish to hedge risk by allocating a smaller fraction of their portfolio to risky assets. Hence for a given equity premium the average risky share- both unconditional and conditional on participation is much lower and closer to what we observe in the data. 2) Wage risk is much larger so that labor income attains stock-like features. In this case wealth-poor agents face a relatively riskier position compared to their wealthy peers since their income consists for the most part of (risky) labor income. To decrease their risk exposure they invest invest relatively more in safe assets. This generates a positive correlation between the risky share and total financial assets. In the PIM workers know their wage profile upon labor market entry. This implies smaller wage risk so that future labor income is a closer substitute to a risk-free bond. In this case wealth-poor agents face a relatively safe position. As they get wealthier their risk exposure increases as their total income depends more heavily on partially risky asset portfolio. To hedge the extra risk wealthier agents allocate a smaller fraction of their savings to stocks. This explains why the PIM predicts a strongly negative correlation between the risky share and total financial assets. 3) Younger investors face a large amount of wage risk. This take place for two reasons. The first relates to the learning process. Early in the life cycle agents face uncertainty about their wage profile.

As agents grow older uncertainty is gradually resolved through the information update. The second reason relates to the time horizon. If wages are risky then young workers are endowed with more stock-like assets than older people. Naturally they allocate a smaller fraction of their savings to risky assets. In the PIM the time horizon effect works in the opposite way (obviously there is no information update). If wages are relatively safe then young workers are endowed with more bond-like assets. Hence they can afford taking risk early at the life cycle since they can always cover any losses. As they approach retirement they compensate the shrinkage in wage stream by rebalancing their portfolio towards safe assets. This mechanism has been analyzed extensively by Jaganathan and Korcelakota (1997) and Cocco, Gomes and Maenhout (2005) and explains why in the PIM the (conditional) risky share of financial assets decreases monotonically along the life cycle.

Apart from contributing to the long literature of portfolio choice this paper is also related to the issue of earnings variation. Asset allocation between stocks and bonds conveys useful information about individuals' perception of their labor income risk. In this sense our paper contributes to the discussion started by Guvenen 2008 and Guvenen and Smith 2010 who use consumption choices to infer how much of wage profile is known at the time of labor market entry. They find that almost 90% of cross sectional variance of wage growth is predictable. In this paper we informally test their findings by expanding the decision set to allocation of savings choices into instruments of different risk. As we have stated the willingness to take financial risk reflects greatly the underlying background risk. Hence estimating individual risk should definitely use information about portfolio choice. Our finings seem to contradict the findings of Guvenen 2008 and Guvenen and Smith 2010. We find that even if people have zero information about their wage profiles we cannot rationalize the low amount of stock holdings at early ages.

Links to the Literature

This paper contributes to the long literature of portfolio choice. The closest paper to ours is the one by Cocco, Gomes and Maenhout (2005) who analyze the portfolio choice allocation along the life cycle. The authors highlight the persistence of the puzzles under a number of different specifications and parameter values. Our work extends their analysis in several important dimensions. First, we introduce imperfect Bayesian learning about labor earnings. This implies that labor earnings are very risky and even more so the younger the investors are. Second, we test our model using a much wider set of statistics. We consider intensive and extensive margin decisions which allows to make a distinction

between conditional and unconditional means. At the same time we also focus on the relation between financial assets and risky share. Since the two are positively related along the life cycle we decompose the risky share across age and wealth groups. On a related paper Gomes and Michaelides (2005) show that heterogeneity in risk aversion and Epstein-Zin preferences cannot bring the model sufficiently close to the data. Ball (2008) examines the portfolio choice under reasonable asset holdings and social security system. He finds that neither the puzzles are robust to the generosity of the social security system. Wachter and Yogo (2010) show that nonhomothetic preferences can provide an explanation for the puzzles.

The most promising in terms of results is the literature that focuses on the correlation between labor income risk and stock returns. Benzoni, Collin-Dufresne and Goldstein (2007) show how at longer time horizons labor income and stock market returns are likely to move together. As a result, in their model stocks are much riskier for young workers than for old workers. Using this mechanism they can generate limited participation in the stock market and increasing age profile of risky share. Storesletten, Telmer and Yaron (2004) argue that idiosyncratic risk is countercyclical so that human capital becomes risky. They obtain a hump shaped investment profile. Lynch and Tan (2010) show that the countercyclical variation in volatility of labor income growth, plays an important role in portfolio choice. By calibrating the first two moments of labor income growth to match the countercyclical volatility and procyclical mean found in U.S. data they can reduce the stock holdings by low wealth young agents. Similar to the above papers we claim that labor income attains stock-like features especially for younger agents. However our rational stands on imperfect information. Although we did not dismiss these ideas as potential explanations of the puzzles we see our approach as a promising complement.

The paper is organized as follows. Section 2 reports some key statistics for the Survey of Consumer Finances about portfolio choice. Section 3 develops a stylized simple model to explain how the basic mechanisms at work. Section 4 illustrates the full model. Section 6 describes the parametrization of the model. Section 7 displays the policy rules the results from our benchmark. Finally, Section 8 concludes.

2 Data-Facts

To examine the basic features of the households' portfolios we look at data from the Survey of Consumer Finances. The study provides information on the structure of households' characteristics and investment decisions. The survey is conducted every three years but we only use statistics from 1998. The basic facts emerging from the analysis are:

- Nearly half of the population does not own some form of a risky account.
- The percentage of investors holding a risky account increases up to retirement.
- For people who invest in some form of risky investment, the risky share of financial assets weakly increases along the life cycle.
- Wealthier people tend to allocate a larger fraction of their savings towards risky accounts.

Definitions The first step to understand households' portfolios is to group assets by the degree of riskiness. To simplify the empirical analysis and link our findings to a portfolio choice model we allocate assets into two categories, namely "safe" and "risky" assets. Some assets can be easily allocated to one of these groups. For example, checking and savings accounts are clearly riskless investments while direct stock holdings clearly involve some risk taking. However, other accounts are invested in a bundle of risk-free and risky instruments. These involve mostly mutual funds and retirement accounts. Fortunately, the Survey of Consumer Finances provides information on how these accounts are invested. The respondents are asked not only how much money they have in an account but also about the way these money are invested. To this end we break these type of accounts into subgroups of "safe" and "risky" type depending on the respond. If (s)he reports that most of the money are in bonds, money market accounts or other risk-free instruments we categorize the account as a risk-free. If the money are invested in some form of stocks we categorize the account as risky. Most of the accounts involve investments in both risk free and risky instruments in which case we assign half of the money into each category.

The assets considered as safe are checking accounts, savings accounts, money market accounts, certificates of deposit, cash value of life insurance, U.S. government or state bonds, mutual funds invested in tax-free bonds or government backed bonds, trusts and annuities invested in bonds, money market accounts, or life insurance and finally pension

plans. On the other hand assets like stocks, brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate and pension plans that are a thrift, profit sharing or stock purchase plan are considered risky. In Table 1 we report the average amount (in '98 dollars) held in specific accounts as well as the portion of the population who holds some positive amount of dollars in this account. We restrict the sample to those who have at least some positive amount of assets. One thing stands out: people show a strong preference towards holding safe assets. While 86.9% of people hold a checking account and 60.2% holds a savings account, only 20.6% own directly stocks. At the aggregate nearly everyone owns some form of safe asset while less than half of the population holds some form of a risky instrument.

Table 1: **Basic Accounts**

Type of Account	Average($)	Participation
Checking account	3,483$	86.9%
Savings account	4,746$	60.2%
Savings bond (safe)	5,916$	22.4%
Life insurance	8,955$	31.7%
Retirement Accounts (safe)	12,103$	38.1%
Total safe assets	64,126$	99.8%
Stocks	32,831$	20.6%
Trust (risky)	6,121$	1.2%
Mutual fund (risky)	12,574$	16.3%
Retirement Accounts (risky)	26,376$	45.2%
Total risky assets	89,336$	56.7%
Total financial assets	153,462$	100%

Portfolio choice by age groups In Figure 1 we display three main statistics for the portfolio choice by age groups. First, the participation decision- the fraction of investors who hold at least one risky asset. Second, the risky share of financial assets- the fraction of total assets allocated towards risky accounts. We document this statistic for the whole population (unconditional share) and for the set of investors who hold at least some positive amount of risky assets (conditional share). Naturally, the conditional share is at least as large as the unconditional share since it excludes from the sample investors with zero money in risky accounts. Risk taking behavior changes significantly with age. Younger investors are more reluctant to hold risky assets than older. On average 40% of individuals at their 20's hold a risky asset. Participation peaks between the age of 51-60,

where on average 65% of people hold a risky account, and decreases after the age of 60. The risky share of financial assets naturally follows this pattern. Individuals at their 20's invest on average 18% of their total assets in risky accounts while individuals between the age of 51-60 invest on average 34%. More interesting is the evolution of the risky share for those investors who hold at least one risky account. By analyzing both decisions we can distinguish the factors responsible for owing risky accounts (extensive margin decision) and fraction of savings allocated in risky instruments (intensive margin decision). The conditional risky share is weakly increasing along the life cycle from nearly 40% at early stages to 50% closer to retirement.

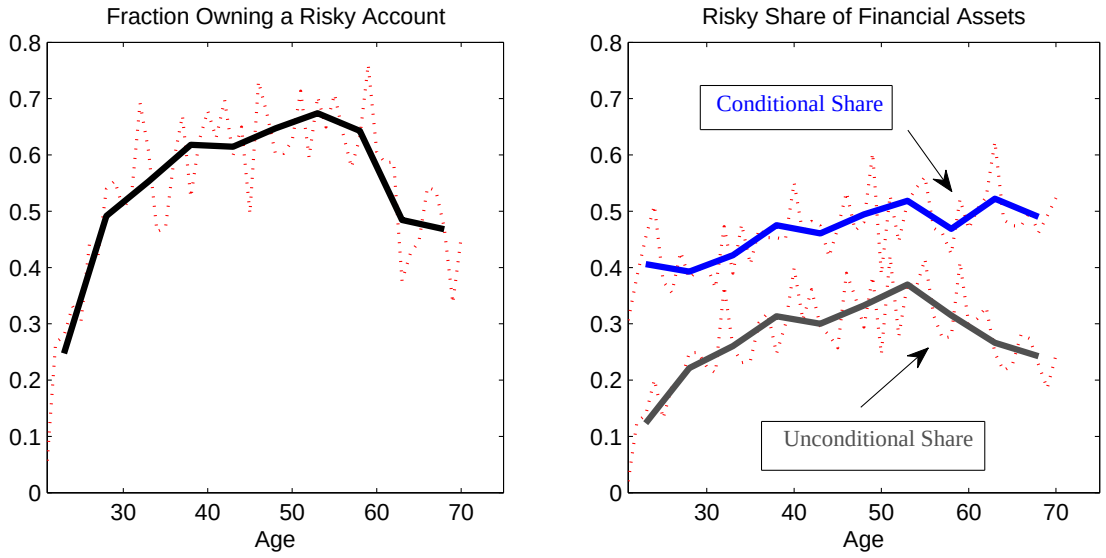

Figure 1: *Left Panel:* Fraction owing a risky account across ages. *Right Panel:* Risky share of financial assets across ages: unconditional sample and sample conditional on owing a risky account.

Portfolio choice by wealth groups We next turn our attention to the relation between the risky share of financial assets and the total amount of financial assets. To this end we decompose the sample across wealth percentiles and calculate the average risky share within percentile. In Figure 2 we plot our findings for the whole sample (left panel) and for the sample conditional on owing a risky account (right panel). The risky share is strongly correlated with total financial assets. Wealthier investors hold a higher amount of their savings in risky accounts than wealth-poor investors. Risky share increases from nearly 15% at the first quartile to nearly 45% at the third. The conditional risky share is

also increasing in financial assets although not as much. Interpreting these patterns is not trivial especially if investors have preferences that exhibit constant relative risk aversion...

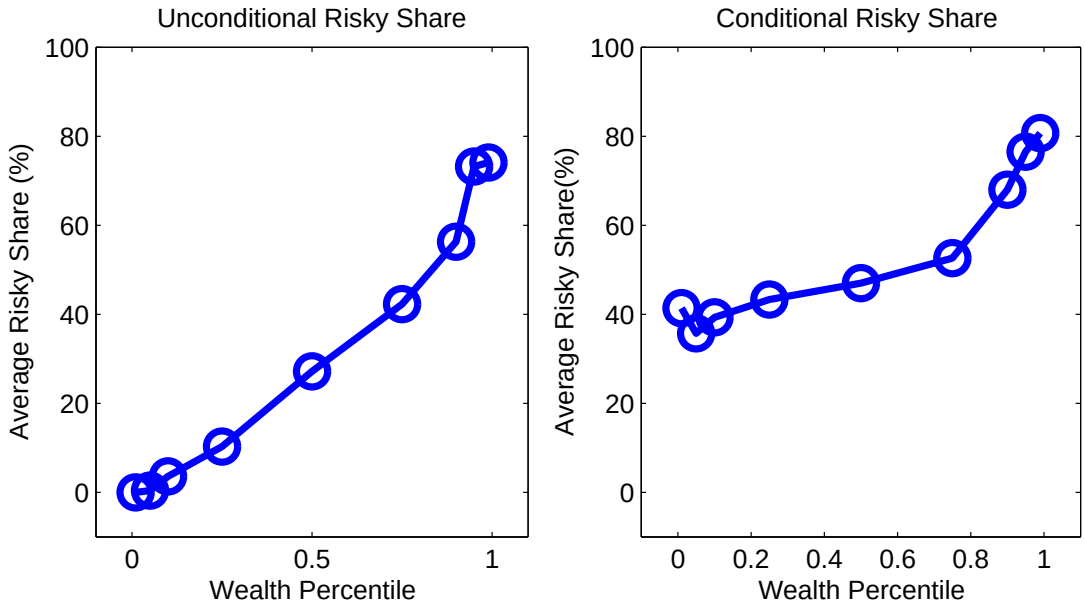

Figure 2: *Left Panel:* Average risky share across wealth percentiles - unconditional sample. *Right Panel:* Average risky share across wealth percentiles - conditional sample.

Econometric Analysis We next estimate jointly the relation between risky share, age and financial assets using a Tobit regression model.

$$risky\ share = \beta_0\, X + \sum_{j=1}^{4} \beta_j \ \ age_{20+10i-30+10i} + \beta_5 \log assets + e$$

The dependent variable is the risky share of financial assets. Regressor X includes several variables like education, sex, marital status, number of children and total income. We also include dummy variables for each age group: 21-30 (omitted), 31-40, 41-50, 51-60 and 61-70. Lastly we include the log of total financial assets. The results from the regression are given in the Appendix. Controlling for financial assets, income and demographic characteristics ages after 30 invest a higher share of financial assets to risky accounts while ages after 60 almost the same. Specifically, ages 31-40 invest on average 9.0 percentage points more in risky accounts than the base group of 21-30. Ages 41-50 invest on average 6.2 percentage points more and ages 51-60 8.3 percentage points more. Lastly, investors between the age of 61-70 invest 0.5 percentage points less in risky accounts

than the base group of 21-30. Our regression reveals a positive relation between financial assets and risky share. A 1% increase in financial assets increase the risky share by 0.07%. Bertaut and Starr-McCluer (2002) also find that risky shares are hump shaped in age after conditioning on log-assets. They also find a positive relationship between the risky share and the amount of financial assets hold by the investor.

3 Simple Portfolio Choice Theory

In this section we solve two and three period models where agents face labor income uncertainty and split their savings between bonds and stocks. The purpose is to build intuition regarding the basic driving forces of portfolio choice. We relate the risky share of financial assets to the total amount of assets, the time horizon and the wage risk. We provide additional intuition in the Appendix with graphical explanations of the relations.

Model The investor lives for $t = 1, ..., T$ periods. Each period she receives income y_t which is an i.i.d random variable with probability function $f(y_t)$. Preferences are given by

$$U = E \sum_{t=1}^{T} \beta^{t-1} \frac{{c_t}^{1-\sigma}}{1-\sigma}$$

where σ is the coefficient of risk aversion. There are two available savings instruments. A bond b_t paying gross return R after one period and a stock s_t whose gross payoff is stochastic and equals $R_s = R + \mu + \eta$. μ is the risk premium which induces the agent to undertake the risky investment and η is the innovation to excess return which is distributed as $N(0, \sigma_\eta^2)$. We denote the associated probability function as $\pi(\eta)$.

Recursive language The investor divides her current output between consumption c and savings $b' + s'$. To make the problem easier we collapse total wealth into a single state variable $W = bR + sR_s$. We do not allow people to borrow. The problem can be written as follows.

$$V_j(W, y) = \max_{c, s', b'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y') df(y') d\pi(\eta') \right\}$$

$$\text{s.t.} \quad c + s' + b' = y + W$$

$$c \geq 0, \quad s' \geq 0, \quad b' \geq 0$$

No labor income In this simple case, investors want to divide their total future income in constant shares of riskless and risky portions, a property of CRRA preferences. The risky share is given by the Samuelson (1963) rule:

$$\frac{s'}{s'+b'} \approx \frac{1}{\sigma}\frac{\mu}{\sigma_\eta^2}$$

The risky share 1) increases in the risk premium 2) decreases in the risk aversion 3) decreases in the volatility of stock returns. In this case total financial assets and time horizon (or age) are irrelevant for the portfolio decision.[1]

A two period model with labor income: Risky Share and Wealth Future labor income is nothing more than an implicit asset with its own risk characteristics. If labor income risk is small then tomorrow's total income will be for the most part safe. As a result, investors facing low wage risk are more willing to take risk in the financial market than investors with uncertain future wages. The degree of wage riskiness also explains differences in portfolio choices between wealth-poor and wealth-rich investors. If wage risk is small future labor income is a close substitute to a risk free asset. Wealth-poor agents face a relatively safe position since most of their future income consists of the risk-free wage. As agents get wealthier they are naturally exposed to more financial risk. To hedge the extra risk wealthier investors will increase relatively more their bond savings i.e. decrease their risky share of financial assets. The relation between risky share and wealth reverses if wage risk is sufficiently high and the investor perceives wages as an implicit risky asset. In this case the wealth-poor have a relatively risky position. As they get wealthier their risk exposure decreases since a significant part of their income depends on their assets. This allows wealthier investors to take a little more financial risk i.e increase the risky share of financial assets. [2]

These properties can be seen in the left panel of Figure 1. We plot the risky share of financial assets $\frac{s'}{s'+b'}$ as a function of cash in hand $W+y$ for different amount of wage risk. Higher wage risk decreases the risky share of financial assets *for given* amount of cash in hand. Investors use this strategy to avoid extreme risk exposure. Also notice that all policy

[1] That is why we did not specify the length of the time horizon. If there is no labor income the investor behaves every period the same way so that results can be generalized into more than two periods.

[2] Our analysis assumes that preferences for risky shares of *total* income remain constant across wealth levels. This result is linked to constant relative risk aversion.

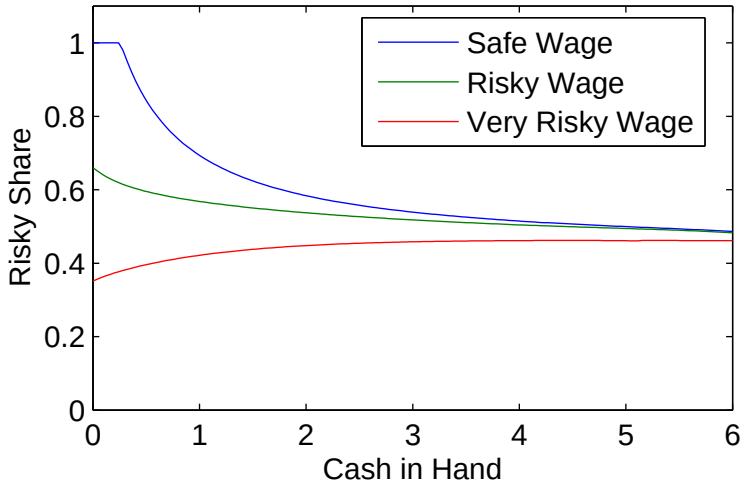

Figure 3: Risky Share and Wealth for different amounts of labor income risk.

functions converge to the same risky share. Intuitively, if assets are very large the wage is only a small part of present discounted income hence wage risk does not matter. The risky share converges to Samuelson rule which corresponds to the zero labor income case. The second thing to notice is that the amount of wage risk also affects the slope of the risky share. If there is no wage risk, the risky share decreases in cash in hand. Wealthier agents increase their bond saving more than their stock savings in order to maintain the same amount of portfolio risk. In contrast high wage risk induces investors to increase their risky share as they get wealthier. Lastly note that the policy function hits the upper bound of one for very small values has to do with the borrowing constraint.

A three period model with labor income: Risky share and time horizon We add one more period in our model to examine the allocations of savings into bonds and stocks across investors of different ages. The second period of this model corresponds to the first period analyzed in the previous two period example. What distinguishes younger from older investors is the total years of future wage payments. If wages are relatively stable then younger investors can afford taking risk in the financial market since they can always use their salaries to cover potential losses. However, as they age the wage stream shortens which induces them to decrease the riskiness of their financial portfolio. On the other hand if wages are sufficiently risky then wage attains stock-like properties. In this case longer time horizon is equivalent to larger risk exposure. As a result younger investors are more conservative in their investment decisions while older investors rebalance their portfolios towards risky assets.

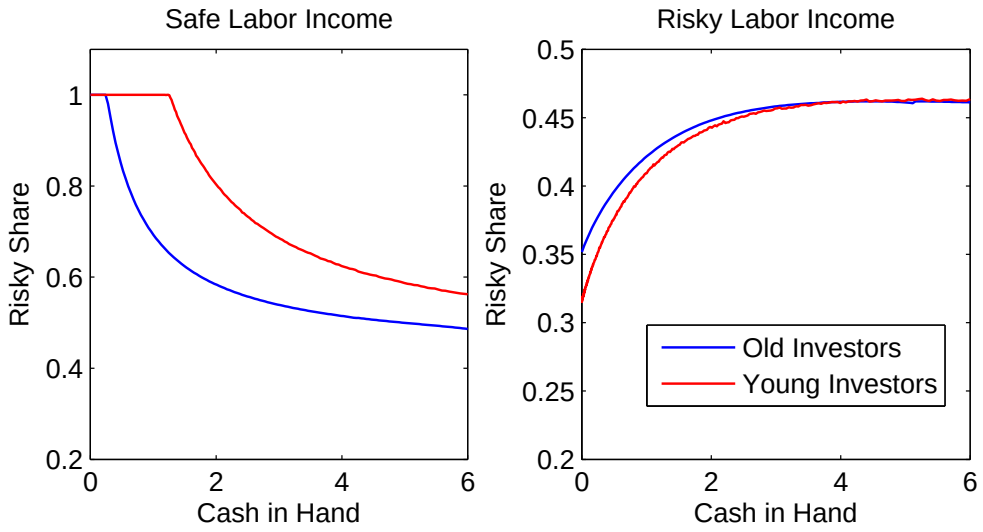

Figure 4: Risky Share and Time Horizon: *Left Panel:* Safe Labor Income. *Right Panel:* Risky Labor Income.

The effect of time horizon on the risky share can be seen in Figure 2. We plot the risky share of financial assets $\frac{s'}{s'+b'}$ as a function of cash in hand $W+y$ for different age groups. The left panel depicts the case of no wage risk while the right panel the case of high wage risk. The policy functions of the old investors correspond to the two period example analyzed before. If there is no wage risk (left panel), younger investors invest relatively more into risky assets. Conditional on cash in hand the risky share is higher. If wage risk is high (right panel) then young investors decrease their risky share of financial assets.

Summary of findings

- All else equal higher labor income risk decreases the risky share of financial assets.
- If wages are relatively safe (risky) wealthy investors decrease (increase) their risky share of financial assets compared to less wealthy investors.
- If wages are relatively safe (risky) younger investors increase (decrease) their risky share of financial assets compared to older investors.

4 Model

The model is a partial equilibrium, life cycle economy. Heterogeneity is expressed in terms of wages, and financial assets both safe (bonds) and risky (stocks). We assume that leisure is not valued so that wages will be exogenous in our model. On the other hand, the distribution of financial assets depends on the investors' endogenous portfolio choice. We develop two versions of the same model. In the first the agent has full information about her labor income process (Perfect Information Model). In the second, she cannot decompose income innovations to its permanent and transitory components (Imperfect Information Model). Our goal is to examine the implications of imperfect information for portfolio choice. The computation techniques to solve the model are described in the Appendix.

Demographics The economy is populated by a continuum of measure one agents (also referred as workers or investors). Every agent enters the labor market at age $j = 1$ and lives for J periods. There is no population growth in the model. At age j_R all agents have to retire. Retirees receive a social security benefit ss which is financed by proportional labor income taxes τ_{ss}.

Preferences Agents derive utility from consumption. Leisure is not valued. Hence, agents devote their one unit of productive time to work. As in our simple example we assume CRRA preferences. Preferences are representable by a time separable utility function of the form :

$$U = E \sum_{j=1}^{J} \beta^{j-1} \frac{{c_j}^{1-\sigma}}{1-\sigma}$$

where β is the discount factor and σ captures both the intertemporal elasticity of substitution and the risk aversion. The literature has focused on alternative preferences to address the portfolio choice puzzles. For example, Gomes and Michaelides (2005) use Epstein-Zin preferences with heterogeneity in both risk aversion and intertemporal elasticity of substitution. Gomes and Michaelides (2005) use habit formation in their preferences. Wachter and Yogo (2008) use nonhomothetic preferences. Our approach is to use standard preferences with constant relative risk aversion. This way we can highlight the potential of labor income risk and learning to solve the portfolio choice puzzles.

Wages Workers face different wages w. Since there is no endogenous labor supply de-

cision wages are equivalent to labor earnings. There are two views regarding the increase in variance of log-earnings along the life cycle. The first supports that it is a result of very persistent idiosyncratic shocks. The second is that it results from heterogeneity in labor income profiles. Following Guvenen (2005, 2007) and Guvenen and Smith (2010) we follow the second strand. In particular, agents face different wage profiles because of differences in initial productivity (fixed effect) The fixed effect component is denoted a and is distributed as $a \sim N(0, \sigma_a^2)$ in the population. We assume that all workers face a common wage growth component denoted γ_j. Workers also experience labor income risk expressed by persistent idiosyncratic wage shocks. These follow an AR(1) process in logs:

$$x_j = \rho x_{j-1} + \nu_j, \qquad \text{with} \quad \nu_j \sim \text{ iid } N(0, \sigma_\nu^2) \tag{1}$$

represented by as a finite state Markov chain $\Gamma(x_j|x_{j-1})$. Lastly, workers face independently distributed idiosyncratic shocks denotes as ε_j. The probability distribution over these shocks is denoted $f(\varepsilon)$. The natural logarithm of wages for an agent of age j is given by

$$w_j = a + \gamma_j + x_j + \varepsilon \tag{2}$$

Asset Markets We assume an incomplete markets model. Agents cannot fully insure against labor income shocks but can partially insure using two savings instruments. A risk free bond b paying gross return R in terms of consumption units after one period. And a stock s which has payoff $R_s = R + \mu + \eta$. μ is the equity premium while η is the innovation to excess return and is distributed as $N(0, \sigma_\eta^2)$. That is on average the stock pays more to convince investors to bear the extra risk.[3] We assume that investors can costlessly rebalance their portfolio each period. In the model savings takes place for two main reasons. First, investors save to prepare for retirement (life cycle savings). Second they save to insure against negative labor income shocks (precautionary savings). At the same time we do not allow investors to borrow. This constraint can be rationalized on the basis of standard moral hazard and adverse selection arguments.

Participation cost To participate in the stock market investors have to pay a fixed monetary cost. A common explanation for these costs is information costs. Stock market participation often requires a significant amount of time to monitor firms' performance, analyze stock market trends and acquire information about growth prospects. At the same time, most portfolios are handled by intermediaries who demand monetary payments.

[3]We do not allow innovations to be correlated with the component of permanent labor income.

The fixed cost is denoted FC.

Government The government engages only in one activity: it runs a balanced social security system. The tax rate τ_{ss} is set to assure the system's budget balance. In the presence of the social security system investors expect a stream of certain labor income, that is, independent of their past labor shocks, once they reach retirement. This allows them to take more risk throughout their life cycle especially as they approach retirement.

Information Structure We will consider two versions of the same model. In the first version agents can identify income innovations to each component separately. We call this a perfect information model (PIM). Wage risk is connected to residual variations of x and ε. The second version is an imperfect information model (IIM). Agents can observe the wage w but cannot identify which component drives the realization. In this case wage risk is connected not only to the residual components but to the wage profile itself. Investors use the wage signals to form more accurate predictions about the unobservable components. In both models the common growth component γ is assumed to be observable.

Investor's problem - Perfect Information Model In this case the investor can observe each wage component separately. The problem is described in a recursive language. As in the simple example we collapse bond and stock holdings in one state variable $W = bR + sR_s$ representing total financial assets. Perfect information implies that each wage component the fixed effect a, the transitory component x and the i.i.d component ε should be included as part of the state vector. The j period value function is denoted as $V_j(W,a,x,\varepsilon)$. Every period investors divide their total income $(w+W)$ between consumption (c) bond savings (b') and stocks savings (s'). After the retirement age they don't have any labor income but they do receive a social security benefit ss.

$$V_j(W,a,x,\varepsilon) = \max_{c,s',b'} \left\{ u(c) + \beta \int_{\eta'} \int_{x'} \int_{\varepsilon'} V_{j+1}(W',a,x',\varepsilon') df(\varepsilon') d\Gamma(x'|x) d\pi(\eta') \right\} \tag{3}$$

$$\text{s.t.} \quad c + s' + b' \ = (1-\tau_{ss}) e^{w_j+\gamma_j} + ss\{j \geq j_R\} - FC\{s' > 0\} + \ W \tag{4}$$

$$w_j = a + \gamma_j + x_j + \varepsilon \tag{5}$$

Equation (4) is the worker's budget constraint if she participates in the stock market. Labor income net of social security contributions and current wealth is divided among consumption and stock savings and bond savings. If stock savings are positive the investor has to pay the fixed cost FC. Retired investors receive only the social security benefit. Individual type a remains constant along the life cycle. The persistent component x' evolves based on the transition matrix $\Gamma(x'|x)$ while the iid component ε based on $f(\varepsilon)$. Next period's income is stochastic both because of the innovations ν and ε. Second, next period's wealth W' is stochastic because the stock return R_s is random. Of course the investor can endogenously decrease her total income riskiness by re-balancing her portfolio over bond holdings.

Investors' problem - Imperfect Information Model In the imperfect information case the wage components cannot be separately identified. Formally, the vector $\mathbf{S}_j = (a, x_j)$ is incompletely known through some noisy signals w_j where

$$w_j = a + x_j + \varepsilon = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ x_j \end{bmatrix} = \mathbf{H}_j' \mathbf{S}_j + \varepsilon_j \tag{6}$$

and through some knowledge of the dynamic evolution of the system

$$\mathbf{S}_{j+1} = \begin{bmatrix} a \\ x_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} a \\ x_j \end{bmatrix} + \begin{bmatrix} 0 \\ \nu_{j+1} \end{bmatrix} = \mathbf{F}\mathbf{S}_j + \mathbf{U}_{j+1} \tag{7}$$

The first equation links the noisy signals with the state variables while second set of equations are used to predict the next period state variables given today's state. The problem is to obtain the best possible estimate of next period's state variables given all available information.

Belief is nothing more than a probability distribution. The agent has prior beliefs about the fixed effect summarized by the first two moments $\{\mu_a, \sigma_a^2\}$. Similarly, the agent has prior beliefs about the AR(1) shock summarized by $\{\mu_x, \sigma_x^2\}$. Beliefs over the cross correlation of the components are given by $\sigma_{a\beta}, \sigma_{\beta x}$. In matrix form:

$$\mathbf{M}_{j|j-1} = \begin{bmatrix} \mu_a \\ \mu_x \end{bmatrix}_{j|j-1} \qquad \mathbf{V}_{j|j-1} = \begin{bmatrix} \sigma_a^2 & \sigma_{ax} \\ \sigma_{ax} & \sigma_x^2 \end{bmatrix}_{j|j-1} \tag{8}$$

Henceforth, the subscript $j|j-1$ will denote prior beliefs about a variable at age j before signal w_j is realized. Similarly, the subscript $j|j$ will denote posterior beliefs about the variable that use the signal w_j as information. The updating occurs in a Bayesian fashion.

In particular posterior means are given by

$$\mathbf{M}_{j|j} = \mathbf{M}_{j|j-1} + \mathbf{G}(w_j - \mathbf{H}_j'\mathbf{M}_{j|j-1}) \tag{9}$$

The posterior belief is the prior belief plus a term that depends on the difference between the signal (w_j) and the agent's belief about her wage before the realization ($\mathbf{H}_j'\mathbf{M}_{j|j-1}$). If her current wage is higher than what she expected she will revisit her beliefs upwards and vice versa. The speed of updating depends on the weight $\mathbf{G}$. [4] The higher the noise variance σ_ε^2 relative to the prior variances σ_a^2 and σ_x^2 the more the agent will discount new information. In this case the updating will be slower. The posterior variance is given by

$$\mathbf{V}_{j|j} = \mathbf{V}_{j|j-1} - \mathbf{AGH}_j'\mathbf{V}_{j|j-1} \tag{10}$$

Note that the variance does not depend explicitly on the wage realization. Successive accumulation of signals decrease the variance. This element introduces information heterogeneity in the model with older-experienced cohorts facing less uncertainty than younger cohorts. An increasing age profile of the risky share would be hard to be justified if investors found out their type quickly. Parameter λ determines the speed of learning. The matrix

$$\mathbf{A} = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

premultiplies the weight $\mathbf{G}$. We rationalize this parameter on the basis of general and firm specific human capital. An agent entering the labor market is more likely to learn quickly about his ability to perform well within the firm she works. In contrast, her general abilities require a series of observations. Parameter λ captures this fact. Mechanically the parameter controls the evolution of both the mean and the variance of the distribution of the fixed effect.

The next step is to use the posterior distributions to forecast next period's income. The agent will enter period $j+1$ having as prior beliefs

$$\mathbf{M}_{j+1|j} = \begin{bmatrix} \mu_a \\ \mu_x \end{bmatrix}_{j+1|j} = \begin{bmatrix} \mu_a \\ \rho\mu_x \end{bmatrix}_{j|j} \tag{12}$$

[4] In particular the term will be equal to $\mathbf{G} = \begin{bmatrix} \frac{\sigma_a^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \\ \frac{\sigma_x^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \end{bmatrix}$.

$$\mathbf{V}_{j+1|j} = \begin{bmatrix} \sigma_a^2 & \sigma_{ax} \\ \sigma_{ax} & \rho^2\sigma_x^2 + \sigma_v^2 \end{bmatrix}_{j|j} \tag{13}$$

Using the above information next period's income will be distributed as

$$F(w_{j+1}|w_j) = N(\mathbf{H}'_{j+1}\mathbf{M}_{j+1|j}, \mathbf{H}'_{j+1}\mathbf{V}_{j+1|j}\mathbf{H}_{j+1} + \varepsilon_j) \tag{14}$$

The state variables are current income w_j, and the probability distributions over the vector $\mathbf{S}_j = (a, \beta, x_j)$. These distributions are described by the first two moments $\mathbf{M}_{j|j-1}$ and $\mathbf{V}_{j|j-1}$. However, we do not have to explicitly include the second moment in the value function since age is a sufficient statistic to describe its evolution.[5] The j period value function is given by

$$V_j(W, w, \mathbf{M}_{j|j-1}) = \max_{c,s',b'} \left\{ u(c) + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y', \mathbf{M}_{j+1|j}) dF_j(y'|y) d\pi(\eta') \right\}$$

$$\text{s.t.} \quad c + s' + b' \;\; = (1 - \tau_{ss}) e^{w_j + \gamma_j} + ss\{j \geq j_R\} - FC\{s' > 0\} + \;\; W \tag{15}$$

The probability distribution $F_j(w'|w)$ is given by (14). Both the current signal w_j and the current belief $\mathbf{M}_{j|j-1}$ form an expectation about next period's income $\mathbf{H}'_{j+1}\mathbf{M}_{j+1|j}$. The dispersion of values along that mean depends on the variance $\mathbf{V}_{j|j-1}$ i.e. on the number of signals accumulated up to age j. Note that beliefs affect current decisions only through changing expectations about tomorrow's income. The problem is solved as follows. Using her signal and her beliefs the agent updates the priors based on (9) and (10). The posterior distributions describe next period's states based on (12) and (13). Finally the expectation about next period's income is given by (14).

5 Calibration

This section discusses the parametrization of the model. There are four set of parameters. Life cycle parameters $\{j_R, J, \tau_{ss}\}$, preference parameters $\{\sigma, \beta\}$, parameters relating

[5] This will be true if agents begin the life cycle as having the same beliefs. We assume this in the simulation.

to the asset market structure $\{\mu, \sigma_\eta^2, r^b, FC\}$ and parameters that govern the labor income process $\{\rho, \sigma_a^2, \sigma_\varepsilon^2, \sigma_v^2, \gamma_j, \lambda, \mathbf{V}_{1|0}\}$.Unless stated otherwise we keep the parameter values constant across the two models.

The model period is one year. The agent lives for a total of $J = 60$ periods. This life framework corresponds to ages 21-80. We assume that agents have to retire at the age of 65. This means that $j_R = 45$. The social security tax is set at $\tau_{ss} = 10\%$. Turning our attention to the preference parameters we set the coefficient of risk aversion $\sigma = 6$. This parameter is an important determinant of the risky share. High risk aversion deters investors from saving into risky assets. We use a value of $\sigma = 6$.

Matching empirically the average level of assets is key to generate a realistic mean of risky share. As mentioned in a number of papers (Storesletten et al. (2004), Ball (2008)) if the average asset holdings is very high wage risk becomes insignificant since labor income becomes a small part of total income. In this case people can use their assets to smooth consumption. Looking back at Figure 2 for sufficiently large assets the risky share converges to its long run value given by the Samuelson rule. Hence having a meaningful distribution of assets is important to make credible predictions about the portfolio choice. Our calibration target is the average financial assets to average wages ratio as found in the SCF '98. We target a capital-income ration equal to 3 as common in the literature. Lastly, we note that we need to recalibrate the parameter for the two models since the amount of wage riskiness is different. In IIM the agents have a much stronger precautionary motive than the PIM. We balance this difference by setting a lower value of β for the former.

Following Cocco, Gomes and Maenhout (2005) we set the equity premium equal to $\mu = 4\%$. This moderate value is common in the literature and represents the equity received by the investors net of taxes. The risk free rate is set to $R = 1.02$ based on the average real rate of US 3-month treasury bills in the post war period. The standard deviation of the innovations to the risky asset is set at 0.18 based on Gomes amd Michaelides (2005). To calibrate the fixed cost of participation we target the average participation rate as found in the SCF equal to 0.56. We find a value $F = 0.03$ i.e 3% of average total labor income. If the average household receives 35,000$ annually in wages the the fixed cost is equivalent to a monetary payment of 1050$. The reader should have in mind two things. This value reflects an average of payments done by the stockholders and second, it reflects more things than just transaction cost like time to monitor the stock market. However the two models have different implications about the calibrated value of the fixed cost. In PIM

there is much more willingness for younger agents to participate than the IIM so we need a much larger value to convince them to stay out of the market.

The last set of parameters involves the labor income process as well as the prior beliefs of the investor. The fixed effect is distributed as $N(0,\sigma_a^2)$. We choose to use two types (a_1,a_2) with equal mass. That gives $a_1 = e^{-\sigma_a}$ and $a_2 = e^{\sigma_a}$. The common wage growth component γ_j is deterministic. Regarding the idiosyncratic productivity component we make the assumption that agents start their careers at the average productivity. The evolution of this component depends on the transition matrix $\Gamma(x'|x)$ which we discretize into a five state Markov chain. The bounds of the x grid are given by $[-3*\frac{\sigma_v}{\sqrt{1-\rho^2}}, 3*\frac{\sigma_v}{\sqrt{1-\rho^2}}]$ where σ_v is the standard deviation of the idiosyncratic innovation to x and ρ represents the persistence of the process. Finally the iid component is distributed as $N(0,\sigma_\varepsilon^2)$ and we use two types $(\varepsilon_1,\varepsilon_2)$ with equal mass. This gives $\varepsilon_1 = e^{-\sigma_\varepsilon}$ and $\varepsilon_2 = e^{\sigma_\varepsilon}$. This gives us a total of 6 parameters to calibrate. We first estimate the common growth component using data from the PSID for periods 1970-1999. More details are given in Appendix 2. For the rest of the parameters our target is to reproduce the cross-sectional variance in log-hourly wages across age cohorts. Again we calculate this profile using data from the PSID. Our findings are consistent with many papers reporting a linearly increasing profile. We find the profile starting at 0.23 at age 22 and reaching 0.56 at age 60. We follow Storesletten et al. (2001) and attribute the cross sectional variation in earnings among the youngest households to a distribution of abilities (fixed effects). That is σ_a^2 is set to 0.21 to match the variance of log earnings upon labor market entry. The residual variation is explained by the variance of the iid component which is set to $\sigma_\varepsilon^2 = 0.06$. We use the linear slope of the log-earnings profile as a targeting moment for $\rho = 0.985$. Finally the variance of the transitory component is set at $\sigma_v^2 = 0.03$ a value used by Storesletten et al. (2004)

It is less clear how to determine the amount of prior beliefs of the investor. The empirical distribution of the fixed effect provides an upper bound for the amount of uncertainty faced by the individual upon entering the labor market. At the same time it would be reasonable to assume that although uncertain the investor still knows more about her type than the econometrician. However, in order to understand the relative strength of the learning mechanism we choose the priors based on the empirical distributions.

$$\mathbf{V}_{1|0} = \begin{bmatrix} 0.21 & 0 \\ 0 & 0.035 \end{bmatrix}_{j|j} \tag{16}$$

6 Quantitative Results

In this section we summarize our results. We first explain the properties of the policy functions under perfect and imperfect information. We then simulate the models and generate key statistics like the participation rate, the average risky share, the age-profile of the risky share and the correlation between risky share and financial assets. We discuss the potential of the two models to explain the empirical patterns discussed in Section 2.

7.2.1 Policy functions First the risky share is significantly lower in the imperfect information model. In the perfect information model risk arises from stochastic fluctuations in wages. In the imperfect information model individuals face a large amount of risk arising both from stochastic fluctuations in wages and uncertainty about the wage profile itself. Lets think of a high skill individual who experiences a temporary low wage maybe because of luck. In the PIM the individual recognizes the nature of the shock and expects her income to bounce back within the next couple of years. In the IIM the individual cannot tell whether she is having a bad year or she is a low skill worker. Effectively the wage risk she is facing is much larger and her investment behavior will reflect this larger risk. In the IIM agents allocate a smaller fraction of savings to stocks to avoid extreme risk exposure.

Second the risky share is lower for younger ages at the IIM but higher for younger ages at the PIM. In the imperfect information model younger investors are less willing to take equity risk for two reasons. First they are uncertain about their profile which can lead to substantial wage risk. As uncertainty is resolved over time they rebalance their portfolio towards risky assets. Second younger people expect a long stream of labor income. Since in the IIM wage is in risk terms equivalent to a risky asset younger people are implicitly endowed with more stock-like assets than older people. As time horizon shortens their implicit holdings of risky assets decrease which allows them to take more financial risk. On contrary under perfect information the risky share is higher for younger people. Since wage risk is relatively small so investors treat labor income as a close substitute to a risk-free bond. In this case younger people can invest more aggressively in the stock market. Not surprisingly the policy functions tend to converge as agents approach retirement. This happens due to the information update and because shorter time horizon makes wage risk insignificant. At age 65 both policy functions overlap.

Third in the PIM wealthier investors decrease significantly their risky share. The IIM

weakens this correlation especially at early ages. The analysis is similar to the previous points. If wage risk is small (PIM) future labor income is a close substitute to the implicit holding of a riskless asset i.e. the wage attains bond-like features. In this case wealth-poor agents face a relatively riskier position compared to their wealthy peers since their income consists for the most part of (risky) labor income. To decrease their risk exposure they invest relatively less in safe assets. This generates a negative correlation between the risky share and total financial assets. In the IIM wage risk is larger so that future labor income is a closer substitute to a stock. In this case wealth-poor agents are exposed to larger risk than their wealthier peers who hold a larger amount of safe assets. To hedge risk wealth-poor agents allocate a relatively larger fraction of their savings to bonds.

Table 2: **Participation and average risky share**

Statistic	Data	PIM	IIM
Participation	54.4%	64.2%	59.8%
Average risky share (unconditional)	27.4%	75.5%	32.2 %
Average risky share (conditional)	46.5%	85.4%	48.0 %

7.2.1 Simulation We use the policy functions to simulate the investment behavior of 10,000 individuals along the life cycle. To highlight differences in individuals' perception of wage risk we use the same panel of individuals for both models. We focus on two sets of statistics: those related to the participation decision and those related to the composition of portfolios. To abstract from the effects that drive participation we distinguish between unconditional shares and portfolios conditional on participation. As seen in Table 2 both models are able to generate *limited participation* in the stock market. Average participation is matched closely- **54.6%** and **57.3%** in the PIM and IIM respectively as opposed to **54.4%** in the data. This should not come as a surprise as the fixed cost was explicitly targeted to match the average participation rates. Nonetheless the fixed cost seems like a good way to explain why almost half of the population does not invest in the stock market. Table 2 also reports the average risky share of financial assets for those who hold a risky account (conditional) and for the whole population (unconditional). Empirically in spite of the high equity premium investors prefer to allocate a small fraction of their savings to stocks. The unconditional portfolio share is **65%** and the conditional share is **75%**. The PIM highlights the surprisingly large demand for stocks given realistic values of risk aversion, equity premium and stock returns. Investors allocate a large fraction of their savings to stocks- **57%** for the whole population and **46%** conditional on participation.

This puzzling feature is directly related to the *equity premium puzzle*[6]. Once we allow for profile uncertainty (IIM) the demand for stocks decreases significantly in response to larger wage risk. The unconditional share is **32%** and the conditional share is **48%**.

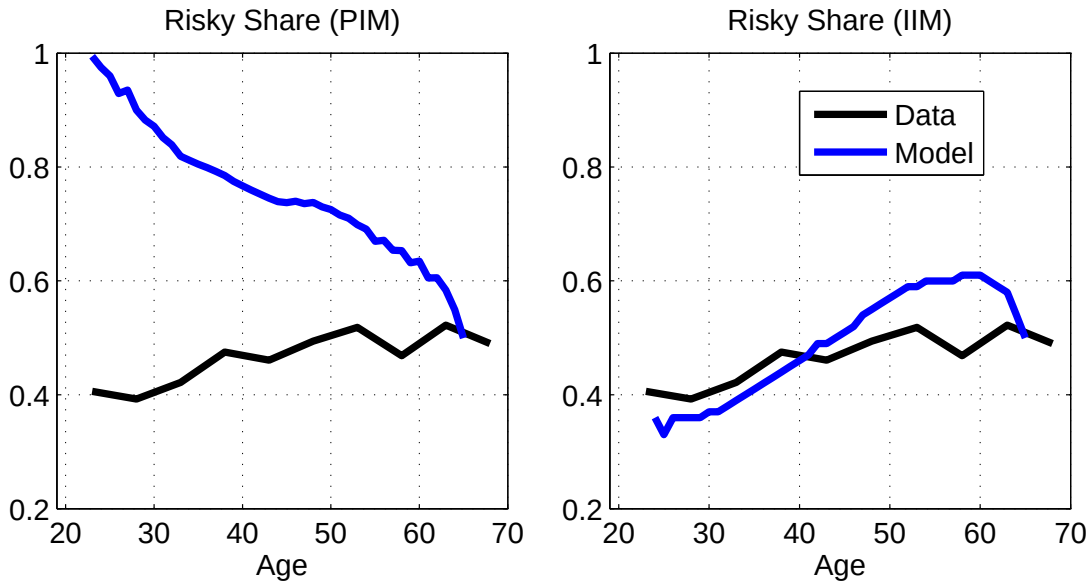

Figure 5: Age Profile of Conditional Risky Share: *Left Panel:* Perfect Information Model. *Left Panel:* Imperfect Information Model.

We next turn our attention to the *portfolio composition puzzles*. Cocco, Gomes and Maenhout (2005) among others document the inability of portfolio choice model to reproduce the weakly increasing evolution of the risky share. This failure is evident in the PIM (Figure 5, left panel) where investors start with a portfolio concentrated in stocks and switch to bond holdings as retirement approaches. The average growth of the risky share is **0.4%** in the data but **-0.2%** in the perfect information version. The underlying mechanism is the small risk exposure of younger cohorts due to a long lifetime of relatively stable wage payments. In sharp contrast the IIM is able to generate an increasing age-profile of risky share (Figure 5, right panel). As discussed before this takes place partially because of the information update and partially due to a longer time horizon of risky payments. The average growth of the risky share is **0.4%**. This is one of the basic results of our paper. Another composition puzzle is the positive correlation between risky share and financial assets **12%** in the data. Figure 6 plots the average risky share for specific

[6]Mehra and Prescott (1986) are the first who showed how the covariance of aggregate consumption with stock returns which defines the riskiness of stock holdings cannot be justified by the high equity premium unless investors are extremely risk averse.

wealth percentiles. IIM seems to outperform the PIM- the correlation is **0.14%** and **-0.50%** respectively. To better convey intuition behind these correlations we turn our attention to the policy functions. There are two features that generate a strongly negative relationship in the PIM: i) the risky share is decreasing in financial assets and ii) older cohorts have lower policy functions than younger cohorts. Since age and wealth are positively related along the life cycle the second effect also contributes to a negative coefficient. A similar analysis explains why IIM succeeds in generating a positive correlation: i) the risky share is decreasing or weakly increasing and ii) conditional on financial assets older (and thus wealthier) people invest relatively more in stocks.

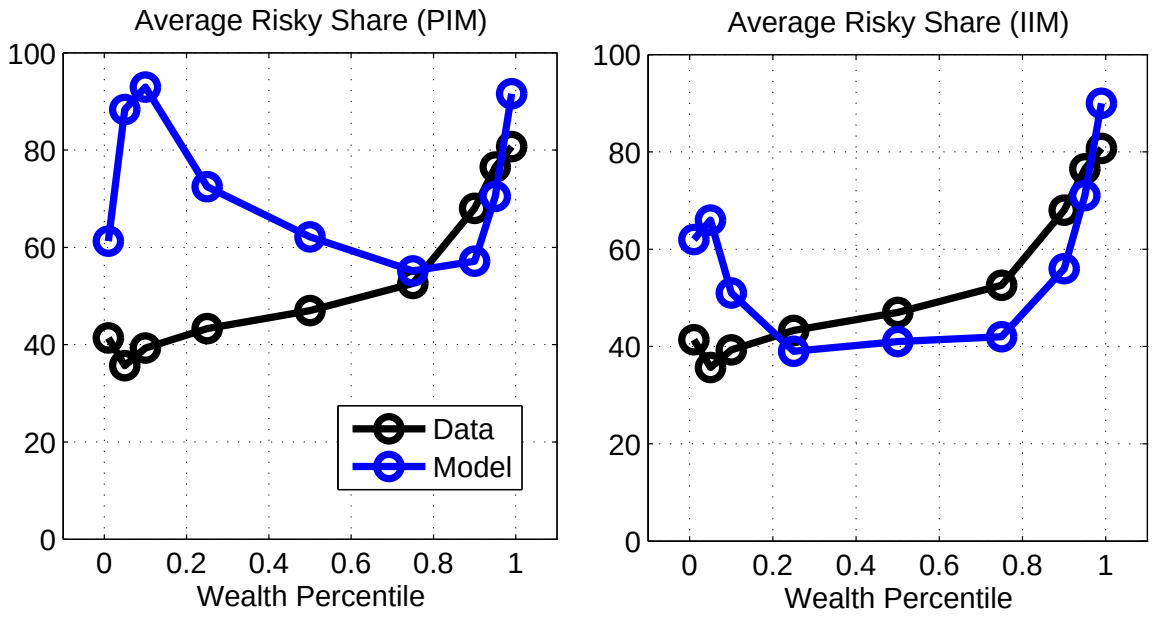

Figure 6: Conditional Risky Share and Wealth Percentiles: *Left Panel:* Perfect Information Model. *Left Panel:* Imperfect Information Model.

7 Conclusion

This paper develops a model of portfolio choice with partial information and optimal learning about labor income. Our main target is to address the main puzzles reported in the literature namely, that one, investors hold a low share of risky assets in their portfolios, two, that the portfolio share of risky assets is larger for the richer people and three the increasing age profile of the risky share. We find that this model can help address these puzzles.

Specifically, higher perceived labor income risk decreases the average risky share. Second, higher perceived risk also induces wealthier agents to increase their risky share since labor income is equivalent to a risky asset. Lastly, optimal learning induces older agents who face smaller uncertainty to buy relatively more stocks. By comparing this model to a model of full information, we find that learning is essential in generating these results.

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Appendix

7.1 A graphical explanation of the risky share decision

We explain here in more detail the relation between (future) labor income and the risky share of financial assets. If labor income is considered relative safe wealthy investors allocate a lower fraction of their savings in stocks, compared to wealth-poor investors. This happens in order to avoid extreme risk exposure. We explain this case using the upper panel of Figure A1 In this case future labor income and bond savings are close substitutes. The investors treats both of them as part of her total future safe income. Because of CRRA preferences both the wealth-poor and the wealth-rich investor expect the same amount prefer their *total future* income to be allocated in constant shares of risky and riskless assets. Lets assume that they prefer 66% of their income in safe and 33% in risky assets. To give specific numbers, assume that future labor income is 1\$ and the poor invests 1\$ in bonds and 1\$ in stocks. Hence her risky share of financial assets is 50%. If the wealthy investor saves twice as much (4\$) she must save 2.33\$ in bond holdings in order to maintain a 66% fraction of *total income* in safe assets. This means that the risky share of the wealthy investor is lower at $\frac{1.66}{4} = 41.5\%$. In the graph the bond savings (green box) has more than doubled. Intuitively the wealthy investor has to decrease her risky share of financial assets otherwise she would be exposed to too much risk. The relation between risky share and wealth reverses if wage risk is sufficiently high. In this case investors feel endowed with a risky asset.We graph this case in the lower panel of Figure 1 in the Appendix. Here, future labor income and stock savings are close substitutes so that they are bundled together by the investor as one risky instrument. Again, we compare the behavior of a wealth-poor and a wealth-rich investor who expect the same amount of future labor income and who both Only for simplicity we assume in this case that both investors prefer their *total future* income to be 33% in safe and 66% in risky assets. Again, to be specific, assume that future labor income is 1\$ and the poor invests 1\$ in bonds and 1\$ in stocks. Hence her risky share is 50%. If the wealthy investor saves twice as much (4\$) she must save 2.33\$ in stocks holdings in order to maintain a 66% fraction of *total income* in risky assets. This means that the risky share of the wealthy investor is higher at $\frac{2.33}{4} = 58.2\%$.

In Figure A2 we relate the risky share to time horizon. What distinguishes younger from older investors is the total years of future wage payments. If wages are relatively stable then younger investors can afford taking risk in the financial market since they can always use their salaries to cover potential losses. However, as they age the wage stream shortens which induces them to decrease the riskiness of their financial portfolio. We compare the behavior of a young and an old investor who save the exact same amount. We assume that in present discounted value terms the young investor has twice the labor income of the old investor. Again we assume that they prefer their *total future* income to be 66% in safe and 33% in risky assets. To give specific numbers, assume

that future labor income is 1$ and the old invests 1$ in bonds and 1$ in stocks. Hence her risky share is 50%. If the young investor has twice as much (2$) in present discounted terms but saves the same amount (2$) she must save 0.66$ in bond holdings in order to maintain a 66% fraction of *total income* in safe assets. This means that the risky share of the young investor is higher at $\frac{1.33}{2} = 66.6\%$. On the other hand if wages are sufficiently risky then shorter working horizon is equivalent to smaller risk exposure. In this case older investors will rebalance their portfolios towards stocks. We assume now that the investors prefer their *total future* income to be 33% in safe and 66% in risky assets. Assume that future labor income is 1$ and the old invests 1$ in bonds and 1$ in stocks. Hence her risky share is 50%. If the young investor has twice as much (2$) in present discounted terms but saves the same amount (2$) she must save 1.33$ in stock holdings in order to maintain a 66% fraction of *total income* in risky assets. This means that the risky share of the young investor is lower at $\frac{0.66}{2} = 33.3\%$.



Fig. A1: Risky share and wealth. *Upper Panel* Safe Labor Income. *Lower Panel:* Risky Labor Income.

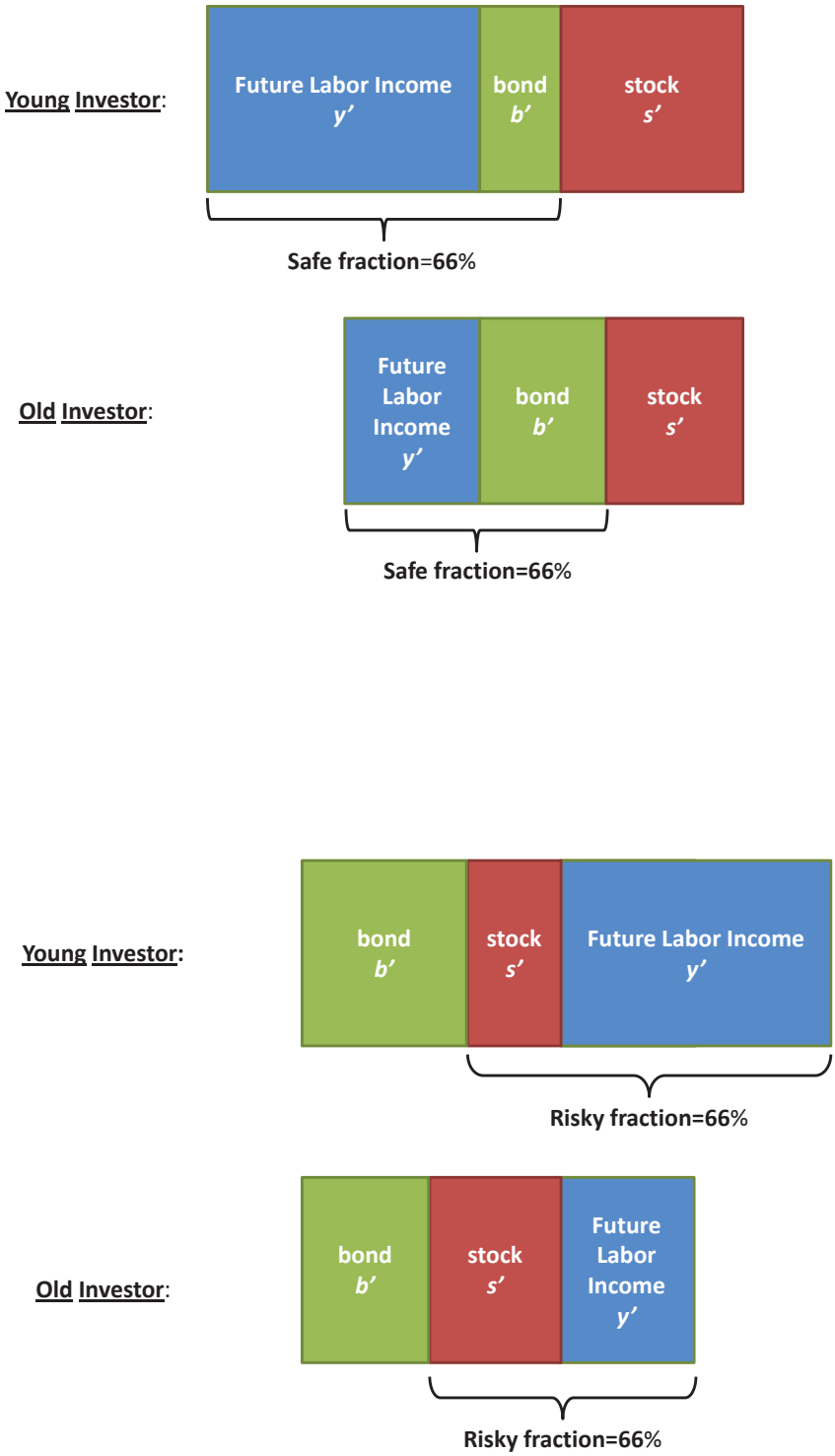

Fig. A2: Risky share and time horizon. *Upper Panel* Safe Wage. *Lower Panel:* Risky Wage.

7.2 Econometric Analysis

Table 3: Econometric Analysis

Explanatory Variable	Coefficient
Education	-.0287
	(.0068)
Male	.0233
	(.0068)
Log Income	.0623
	(.0041)
Married	-.0166
	(.0073)
Children	-.0157
	(.0017)
Age 31-40	.1260
	(.0083)
Age 41-50	.0877
	(.0080)
Age 51-60	.1169
	(.0088)
Age 61-70	-.0079
	(.0103)
Log Assets	.1068
	(.0016)
Observations	21,525

7.3 Calibration

Earnings Process

We use the Panel Study of Income Dynamics (PSID) waves 1971-1999 and collect information on head of households about annual labor income, age, year of study, annual hours worked and sex. We include in the sample households whose head 1) is between 21-63 2) is a male 3) works between 520 and 5096 hours annually 4) hourly wage (equal to annual earnings divided by annual hours) is more than half of the minimum wage for the respective year. Earnings are transformed into 1998 dollars using a price deflator. We use the log of earnings to compute firstly the common wage growth component. We do this by regressing log-earnings on a second order polynomial with age. Results are plotted in the left panel of Figure A4. Secondly we compute the cross sectional variances for every age net of cohort effects. To do so we compute second moments as a function of age and

Parameter	Notation	Value
Risk Aversion	σ	5
Discount Factor: FIM (PIM)	β	1.05 (0.97)
Risk free rate	R	1.02
Equity Risk Premium	μ	0.04
Stock Return Volatility	σ_η	0.18
Transition probability for stock	$\pi(\eta)$	0.5
Social Security Tax	τ_{ss}	0.10
Variance of fixed effect	σ_a^2	0.2173
Variance of transitory component	σ_v^2	0.035
Persistence parameter	ρ	0.985
Variance of iid component	σ_ε^2	0.063
Learning Parameter	λ	0.5

cohort and regress the variances on a full set of age and cohort dummies. The right panel plots the age dummies.

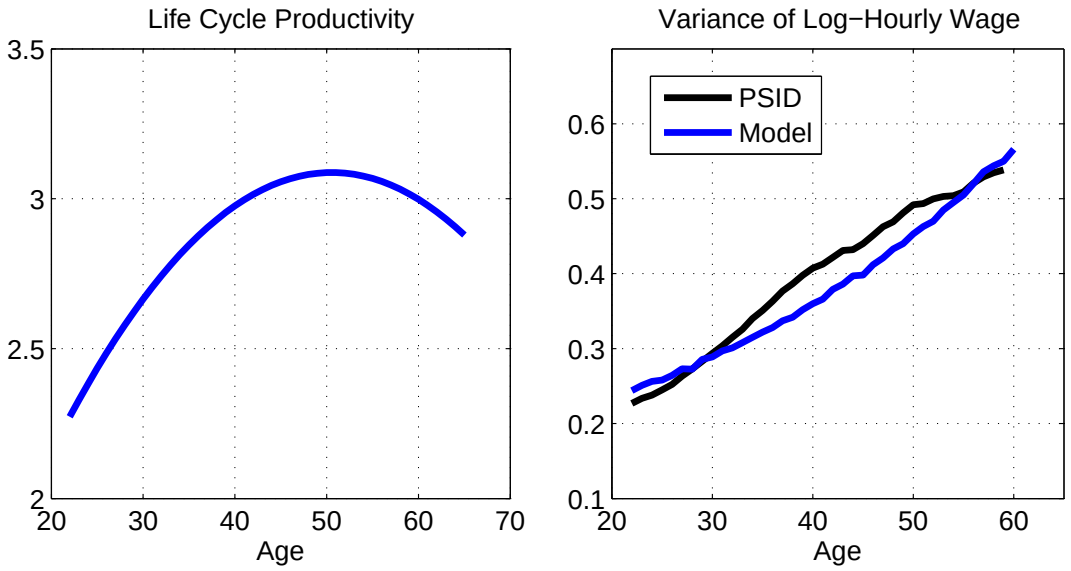

Fig. A3: *Left Panel:* Estimated wage growth. *Right Panel:* Cross-sectional variance of log-earnings net of cohort effects.

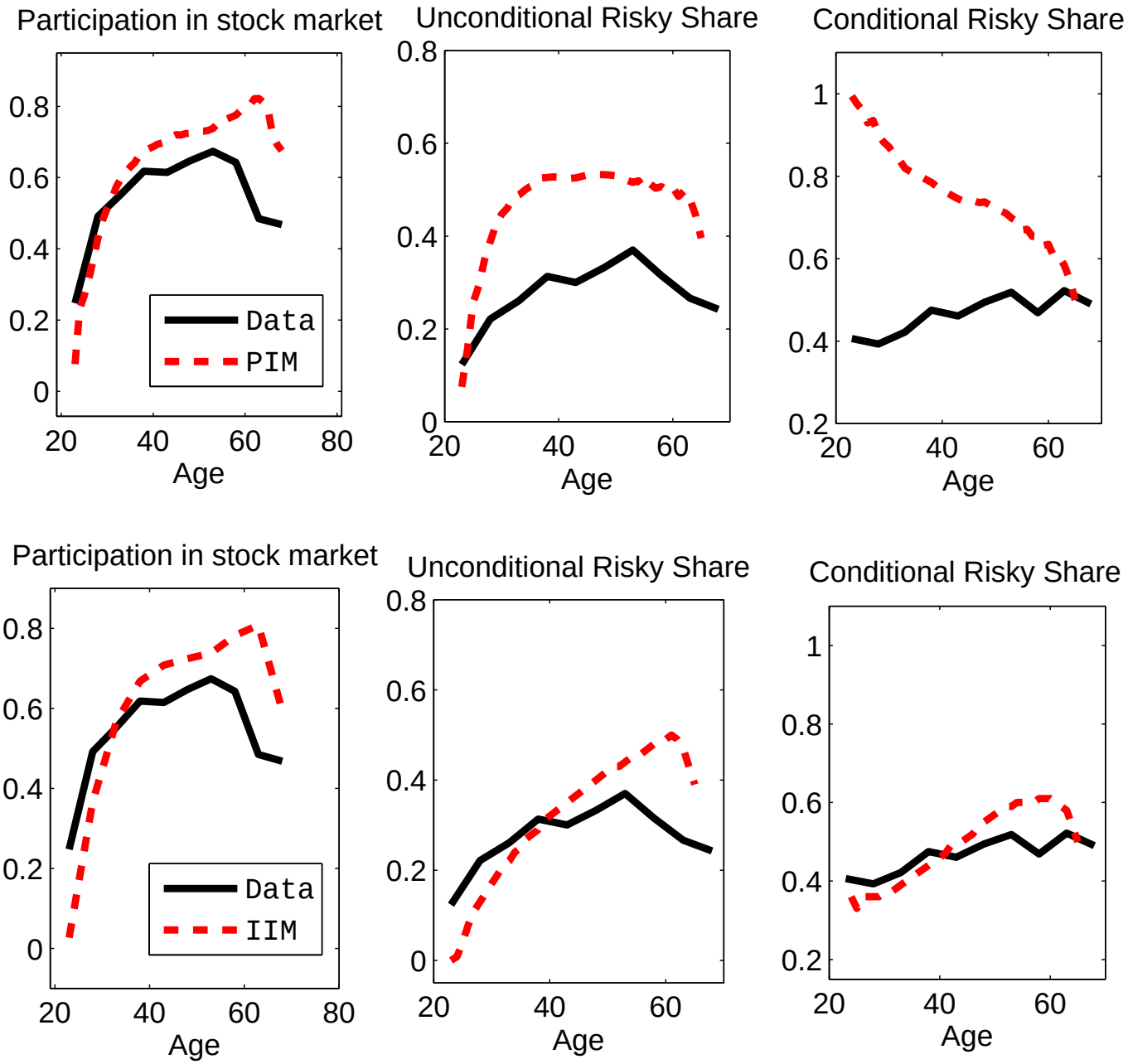

Fig. A4: Data and models' fit for portfolio choice. *Upper Panels:* Perfect Information Model. *Lower Panels:* Imperfect Information model.